Continuous-Time Filters

1.0 Operational Transconductance Amplifier (OTA)

Figure 1. Ideal small signal equivalent circuit of Single ended OTA and Fully differential OTA Implementation Using Single ended OTAs.

Figure 1(a) shows the symbol of single ended OTA. Its ideal equivalent circuit is shown in Figure 1(b). Its operation is given by:

\[ i_o = G_m v_i = G (v^+ - v^-) \]

\[ Z_{in} = \infty \]

\[ Z_{out} = \infty \]
That is, its output impedance is $\infty$ which is 0 for ideal opamp. A fully differential OTA shown in Figure (c) can be implemented using two single ended OTAs with twice the transconductance. Its connection is shown in Figure 1(d). The equivalency can be shown as follows:

\[ I_1 = I_o = 2G_m V_i^+ \]
\[ I_2 = -I_o = 2G_m V_i^- \]
\[ I_1 - I_2 = I_o - (-I_o) = 2I_o = 2G_m (V_i^+ - V_i^-) \]
\[ I_o = G_m (V_i^+ - V_i^-) = G_m V_i \]

2. OTA Simple Building Block Circuits

\[ Z_i = \frac{1}{G_m} \]

Figure 2 OTA simple building blocks

Figure 2(a) shows a simple voltage positive gain amplifier. Its gain is derived as follows:

\[ I_o = G_m V_i \]
\[ V_o = R I_o = G_m RV_i \]
\[ A_v = \frac{V_o}{V_i} = G_m R \]

\[ Z_i = \infty \], since the input impedance of OTA is $\infty$, and $Z_o=R$, since the output impedance of OTA is $\infty$.

In Figure 2(b) is the same as Figure 2(a) except for the input inversion resulting in negative gain. In Figure 2(c),
\[ A_V = \frac{V_o}{V_i} = 1; \text{ since } V_o = V_i \]

\[ I_o = -G_m V_i = -I_i \]

\[ Z_i = \frac{V_i}{I_i} = \frac{1}{G_m} = Z_o \]

Figure 2(d) is the same as in Figure 2(a) with a load \( R = 1/G_{m2} \).

\[ A_V = G_{m1} R = \frac{G_{m1}}{G_{m2}} \]

As in Figure 2(a), the input and output impedance of Figure 2(d) are:

\[ Z_i = \infty; Z_o = \frac{1}{G_{m2}} \]

In Figure 2(e),

\[ I_1 = G_{m1} V_i \]
\[ I_2 = -G_{m2} V_o \]

\[ V_o = Z_L I_1 = Z_L G_{m1} V_i \]

\[ A_V = \frac{V_o}{V_i} = G_{m1} Z_L \]

\[ I_2 = -G_{m1} G_{m2} Z_L V_i = -I_i \]

\[ Z_i = \frac{V_i}{I_i} = \frac{1}{G_{m1} G_{m2} Z_L} \]

That is, the input impedance is the reciprocal of the load impedance. If the load impedance is capacitive, then the input impedance is inductive. This circuit is an impedance converter. Since the output impedance of OTA1 and the input impedance of OTA2 are both \( \infty \), the circuit output impedance is:

\[ Z_o = Z_L \]
These are summarized in the following table.

**Table 1: OTA Simple Building Block Circuits.**

<table>
<thead>
<tr>
<th>Circuit Type</th>
<th>$Av$</th>
<th>$Z_i$</th>
<th>$Z_o$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voltage amplifier with positive gain, Fig 2(a)</td>
<td>$G_mR$</td>
<td>$\infty$</td>
<td>$R$</td>
</tr>
<tr>
<td>Voltage amplifier with negative gain, Fig 2(b)</td>
<td>$-G_mR$</td>
<td>$\infty$</td>
<td>$R$</td>
</tr>
<tr>
<td>OTA Resistance, Fig 2(c)</td>
<td>$1$</td>
<td>$1/G_m$</td>
<td>$1/G_m$</td>
</tr>
<tr>
<td>Voltage amplifier using OTA resistance, Fig 2(d)</td>
<td>$G_{m1}/G_{m2}$</td>
<td>$\infty$</td>
<td>$1/G_{m2}$</td>
</tr>
<tr>
<td>OTA impedance converter, Fig 2(e)</td>
<td>$G_mZ_L$</td>
<td>$1/(G_{m1}G_{m2}Z_L)$</td>
<td>$Z_L$</td>
</tr>
</tbody>
</table>

3. **First Order Filter Building Block Circuits**

![Figure 3. First Order Filter Single-Ended OTA Building Blocks.](image-url)
Figure 3 shows the first order filter building blocks. The two lowpass filter implementations in Figure 3(a) and 3(b) will be shown to be the same. Also, only Figure 3(a) can be converted to fully differential ota implementation shown in Figure 4(a).

In Figure 3(a),

\[ I_1 = G_m V_i \]
\[ I_2 = -G_m V_o \]
\[ I_C = sC V_o = I_1 + I_2 = G_m V_i - G_m V_o \]
\[ (sC + G_m) V_o = G_m V_i \]
\[ \frac{V_o}{V_i} = \frac{G_m}{sC + G_m} = \frac{G_m / C}{s + G_m / C} = \frac{w_o}{s + w_o} \]

In Figure 3(b),

\[ I_o = G_m (V_i - V_o) = sC V_o \]
\[ \frac{V_o}{V_i} = \frac{G_m}{sC + G_m} = \frac{G_m / C}{s + G_m / C} \]

That is, Figure 3(a) and 3(b) implement the same transfer function.

In Figure 3(c),

\[ I_o = -G_m V_o = sC(V_o - V_i) \]
\[ \frac{V_o}{V_i} = \frac{sC}{sC + G_m} = \frac{s}{s + G_m / C} = \frac{s}{s + w_o} \]

In Figure 3(d),

\[ I_1 = G_{m1} V_i \]
\[ I_2 = -G_{m2} V_o \]
\[ I_{C2} = sC_2 (V_i - V_o) \]
\[ I_{C1} = sC_1 V_o = I_1 + I_2 + I_{C2} = G_{m1} V_i - G_{m2} V_o + sC_2 (V_i - V_o) \]
\[ \frac{V_o}{V_i} = \frac{sC_2 + G_{m1}}{s(C_1 + C_2) + G_{m2}} = \frac{s[C_2/(C_1 + C_2)] + [G_{m1}/(C_1 + C_2)]]}{s + [G_{m2}/(C_1 + C_2)]} = \frac{a_1 s + a_0}{s + w_o} \]

5
Table 2: OTA First Order Filter.

<table>
<thead>
<tr>
<th>Circuit Type</th>
<th>Transfer Function</th>
<th>Wo</th>
</tr>
</thead>
<tbody>
<tr>
<td>First order Lowpass, Fig 3(a),(b)</td>
<td>$\frac{G_m}{C}$</td>
<td>$G_m/C$</td>
</tr>
<tr>
<td>First order Highpass, Fig 3(c)</td>
<td>$\frac{s}{s + G_m/C}$</td>
<td>$G_m/C$</td>
</tr>
<tr>
<td>General First order Fig 3(d)</td>
<td>$\frac{s[C_2/(C_1 + C_2)] + [G_{m1}/(C_1 + C_2)]}{s + [G_{m2}/(C_1 + C_2)]}$</td>
<td>$G_{m2}/(C_1 + C_2)$</td>
</tr>
</tbody>
</table>

A universal first order filter can be implemented as shown in Figure 3(e). The universal filter has four terminals $V_i$, $V_o$, $V_1$, and $V_2$. The lowpass filter can be selected by connecting the input $V_i$ to $V_1$ and $V_2$ to ground. The highpass filter is selected by connecting the input $V_i$ to $V_2$ and $V_1$ to ground. These are summarized in Figure 3(f).

---

Figure 4. Differential OTA Implementation of First-Order Filters.
Figure 4 shows the fully differential OTA implementation of the first order filters. It shows the number and value of capacitors doubled. Consequently, it will consume more VLSI real estate. However, a fully differential circuits implementation have the advantage of better noise immunity and distortion properties. The equivalently of the two implementations will be illustrated for the first order lowpass and highpass filters. That is, the transfer function of Figure 4(a) will be shown to be identical to that of Figure 3(a) or 3(b). In Figure 4(a):

\[ I_1 = G_m V_i = G_m (V_i^+ - V_i^-) \]
\[ I_2 = G_m V_o = G_m (V_o^+ - V_o^-) \]
\[ I_{C'} = -2sC V_o^+ = -I_1 + I_2 \]
\[ I_{C''} = 2sC V_o^- = I_1 + I_2 \]
\[ I_{C'} + I_{C''} = -2sC(V_o^+ - V_o^-) = -2I_1 + 2I_2 \]
\[ -sC(V_o^+ - V_o^-) = -I_1 + I_2 = -G_m (V_i^+ - V_i^-) + G_m (V_o^+ - V_o^-) \]
\[ -sC V_o = -G_m V_i + G_m V_o \]
\[ (sC + G_m)V_o = G_m V_i \]
\[ \frac{V_o}{V_i} = sC + G_m = \frac{G_m}{s + G_m / C} \]

The high pass filter shown in Figure 4(b) with one fully differential ota implementation will be shown to have the same transfer function as the single-ended ota in Figure 3(c). The derivation follows:

\[ I_o = G_m V_o = G_m (V_o^+ - V_o^-) \]
\[ I_o = 2sC(V_i^+ - V_o^+) \]
\[ I_o = 2sC(V_o^- - V_i^-) \]

Adding the above two equations and equating with the first equation,
\[ 2I_o = 2sC[-(V_o^+ - V_o^-) + (V_i^+ - V_i^-)] = 2G_m (V_o^+ - V_o^-) \]
\[ sC[-V_o + V_i] = G_m V_o \]
\[ \frac{V}{V} = sC \frac{s}{sC + G_m} = \frac{s}{s + G_m / C} \]

The general first order filter shown in Figure 4(c) can similarly be derived. Note that lowpass filter needs two fully differential otas, while for highpass filter requires only one.

The calculation is independent of whether using lowpass or highpass transfer function. The lowpass filter transfer function will be used for illustration. The transfer function to be implemented is given by:

\[ L1(s) = \frac{W}{s + W} = \frac{W_o}{s + W_o} \]
In all the subsequent examples, it will assumed that the value of \( w = 6.2832 \times 10^5 \) corresponding to a frequency of 100kHz.

The capacitance value for a given \( G_m = 85.44 \mu \) is obtained as follows:

\[
\begin{align*}
C &= \frac{G_m}{w_o} = \frac{85.44 \times 10^{-6}}{6.2832 \times 10^5} = 135.98 \text{pF}
\end{align*}
\]

The general filter transfer function to be implemented has a pole at \( f_p = 100 \text{kHz} \), and zero at \( f_z = 200 \text{kHz} \) and with DC gain of 0.5. That is,

\[
\begin{align*}
T_F(s) &= \frac{0.5(s + 2\pi 2e + 5)}{(s + 2\pi 1e + 5)} = \frac{0.5(s + 2\pi 1e + 5)}{(s + 2\pi 1e + 5)} = \frac{a_1 s + a_o}{s + w_o} = \frac{[C_2/(C_1 + C_2)]s + [G_m/(C_1 + C_2)]}{s + [G_m/(C_1 + C_2)]} \\
C_2 &= a_1 \\
C_2 &= \frac{a_1}{1-a_1}C_1 \\
G_m\frac{1}{C_1 + C_2} &= a_o \\
G_m\frac{1}{C_1 + C_2} &= w_o
\end{align*}
\]

Assume \( G_{m2} \) is given

\[
\begin{align*}
C_1 + C_2 &= C_1 + \left( \frac{a_1}{1-a_1} \right)C_1 = \frac{1}{1-a_1}C_1 = \frac{G_m}{w_o} \\
C_1 &= (1-a_1)\frac{G_m}{w_o} = (1-0.5)\frac{85.44 \times 10^{-6}}{2\pi 1e + 5} = 67.99 \text{pF} \\
C_2 &= \left( \frac{a_1}{1-a_1} \right)C_1 = \left( \frac{0.5}{1-0.5} \right)(67.99 \text{pF}) = 67.99 \text{pF} \\
G_m = a_o (C_1 + C_2) = a_o \frac{G_m}{w_o} = (2\pi 1e + 5)\frac{85.44 \times 10^{-6}}{2\pi 1e + 5} = 85.44 - 6
\end{align*}
\]
3.1 First Order Filters Simulation

First Order Lowpass Filter Transfer Function

*Filename = "lp1.cir"
*First order lowpass filter fo=100k

.PARAM w=6.2832e+5
Vin 1 0 DC 0V AC 1V
R1 1 0 1E+20
E2 2 0 LAPLACE \{V(1)\}={w/(s+w)}
Rout 2 0 1k

* Analysis
.AC DEC 10 1Hz 100MegHz
.PROBE
.END

First Order Highpass Filter Transfer Function

*Filename = "hp1.cir"
*First order lowpass filter fo=100k

.PARAM w=6.2832e+5
Vin 1 0 DC 0V AC 1V
R1 1 0 1E+20
E2 2 0 LAPLACE \{V(1)\}={(s/(s+w))}
Rout 2 0 1k

* Analysis
.AC DEC 10 1Hz 100MegHz
.PROBE
.END

General First Order Filter Transfer Function

*Filename = "gen1.cir"
*General first order filter p=100kHz, z=200kHz

.PARAM wp=6.2832e+5
*.PARAM wz=2*wp
.PARAM wz=12.5664e+5
Vin 1 0 DC 0V AC 1V
R1 1 0 1E+20
E2 2 0 LAPLACE \{V(1)\}={(0.5*(s+wz)/(s+wp))}
Rout 2 0 1k

* Analysis
.AC DEC 10 1Hz 100MegHz
.PROBE
.END
Ideal First Order Lowpass Filter Response

Ideal First Order Highpass Filter Response
Ideal General First Order Filter Response

**Universal First Order Filter Single-ended OTA Implementation**

*Filename = "univ1_i1.cir", see Figure 3e
*Universal First order filter. fo=100K
*Using Single-ended OTA
V1 1 0 DC 0V AC 1V
*LP
*Xs1flt 1 0 2 S1FLT
*HP
Xs1flt 0 1 2 S1FLT
R2 2 0 1E+20

.SUBCKT S1FLT v1 v2 v0
.PARAM C=135.98pF
Xota1 v1 v0 v0 WSOTA
Cl v0 v2 (C) IC=0V
. ENDS

.SUBCKT WSOTA in+ in- out
GI 0 out in+ in- 85.44U
. ENDS

* Analysis
.AC DEC 100 1Hz 100MegHz
.PROBE
.END

**General First Order Filter Single-ended OTA Implementation**

*Filename = "gen1_i1.cir", see Figure 3d
*General First order filter. p=100kHz, z=200kHz
*Using Single Ended OTA
First Order Lowpass Filter Response

```plaintext
.PARAM C=67.99pF
Vin 1 0 DC 0V AC 1V

Xota1 1 0 2 WSOTA
Xota2 0 2 2 WSOTA
C1 2 0 {C}
C2 1 2 {C}
R2 2 0 1E+20

.SUBCKT WSOTA in+ in- out
Gi 0 out in+ in- 85.44U
.ENDS

* Analysis
.AC DEC 100 1Hz 100MegHz
.PROBE
.END
```

First Order Lowpass Filter Response
Universal implementation is not possible using fully differential OTA. Lowpass filters require two fully differential OTAs, while highpass filters only need one.

Fully Differential OTA Implementation of First Order Lowpass Filter
*Filename = "lp1_id.cir", see Figure 4a
*First order filter fo=100K
*Using fully differential OTA

Vin 1 2 DC 0V AC 1V

.PARAM C=135.98pF
Xota1 1 2 3 4 DIFFOTA
Xota2 3 4 4 3 DIFFOTA

C1 3 0 (2*C) IC=0V
C2 4 0 (2*C) IC=0V
R1 1 0 1E+20
R2 2 0 1E+20
R3 3 0 1E+20
R4 4 0 1E+20

* Fully Differential OTA Implementation
* by definition
.SUBCKT DIFFOTA in+ in- out+ out-
G1 0 out+ in+ in- 85.44U
G2 0 out- in+ in- -85.44U
.ENDS

* Analysis
.AC DEC 100 1Hz 100MegHz
.PROBE
.END

Fully Differential OTA Implementation of First Order Highpass Filter

*Filename = "hp1_id.cir", see Figure 4b
*First order filter. fo=100K
*Using Fully Differential OTAs

Vin 1 2 DC 0V AC 1V

.PARAM C=135.98pF
Xota1 3 4 4 3 DIFFOTA
Xota2 1 3 DIFFOTA

C1 1 3 (2*C) IC=0V
C2 2 4 (2*C) IC=0V
R1 1 0 1E+20
R2 2 0 1E+20
R3 3 0 1E+20
R4 4 0 1E+20

* Fully Differential OTA Implementation
* by definition
.SUBCKT DIFFOTA in+ in- out+ out-
G1 0 out+ in+ in- 85.44U
G2 0 out- in+ in- -85.44U
.ENDS

* Analysis
.AC DEC 100 1Hz 100MegHz
.PROBE
.END

Fully Differential OTA Implementation of General First Order Filter

*Filename = "gen_id.cir", see Figure 4c
*First order filter fp=100kHz, fz=200kHz
*Using fully differential OTA
Vin 1 2 DC 0V AC 1V

.PARAM C=67.99pF
Xota1 1 2 3 4 DIFFOTA
Xota2 3 4 4 3 DIFFOTA

C11 3 0 (2*C) IC=0V
C12 4 0 (2*C) IC=0V
C21 1 3 (2*C) IC=0V
C22 2 4 (2*C) IC=0V
R1 1 0 1E+20
R2 2 0 1E+20
R3 3 0 1E+20
R4 4 0 1E+20

* Fully Differential OTA Implementation
* by definition
.SUBCKT DIFFOTA in+ in- out+ out-
G1 0 out+ in+ in- 85.44U
G2 0 out- in+ in- -85.44U
.ENDS

* Analysis
.AC DEC 100 1Hz 100MegHz
.PROBE
.END

First Order Lowpass Filter Response
First Order Highpass Filter Response

General First Order Filter Response
4. Second Order Filter Building Block Circuits

![Diagram of second order filters with fixed Q and 2 OTAs each.](image)

(a) LP(s)  
(b) HP(s)  
(c) BP(s)  
(d) BR(s)

Figure 5. Second Order Filters Implementation with fixed Q and 2 OTAs each.

There are two second order filter building blocks. One requiring two OTAs and the other 3 OTAs. The selection depends on whether an adjustable Q is required or not.
Fixed Q Implementation

Figure 5 shows the second order filter implementation using 2 OTAs. The implementation has a non-adjustable Q, once the capacitive components had been determined. The transfer function derivation will be demonstrated for the lowpass circuit. In Figure 5(a):

\[ I_1 = G_{m1} (V_i - V_o) \]
\[ I_2 = G_{m2} (V_1 - V_o) \]
\[ V_1 = \frac{I_1}{sC_1} = \frac{G_{m1}}{sC_1} (V_i - V_o) \]
\[ V_o = \frac{I_2}{sC_2} = \frac{G_{m2}}{sC_2} \left( \frac{G_{m1}}{sC_1} (V_i - V_o) - V_o \right) \]
\[ \frac{V_o}{V_i} = \frac{G_{m1} G_{m2}}{s^2 C_1 C_2 + sC_1 G_{m2} + G_{m1} G_{m2}} = \frac{G_{m1} G_{m2}}{C_1 C_2} \]

Comparing the transfer function with the standard lowpass transfer function given in Table 1. The \( \omega_o \) and Q are obtained as follows:

\[ \omega_o = \sqrt{\frac{G_{m1} G_{m2}}{C_1 C_2}} = \frac{G_m}{\sqrt{C_1 C_2}} \quad \text{if} \quad G_m = G_{m1} = G_{m2} \]

\[ \frac{\omega_o}{Q} = \frac{G_{m2}}{C_2} \]

Solving for Q,

\[ Q = \frac{\omega_o C_2}{G_m} = \sqrt{\frac{C_2}{C_1}} \]

Note that for a given values of C1 and C2, Q is fixed but \( \omega_o \) can be adjusted by changing \( G_m \). The expressions for the other circuits can be derived in a similar way. The results are summarized in Table 4.
Table 3. Second Order Filter Transfer Function.

<table>
<thead>
<tr>
<th>Circuit Type</th>
<th>Transfer Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lowpass</td>
<td>( \frac{w_o^2}{s^2 + s(w_o/Q) + w_o^2} )</td>
</tr>
<tr>
<td>Highpass</td>
<td>( s^2 )</td>
</tr>
<tr>
<td></td>
<td>( s^2 + s(w_o/Q) + w_o^2 )</td>
</tr>
<tr>
<td>Bandpass</td>
<td>( \frac{s(w_o/Q)}{s^2 + s(w_o/Q) + w_o^2} )</td>
</tr>
<tr>
<td>Bandreject</td>
<td>( \frac{s^2 + w_o^2}{s^2 + s(w_o/Q) + w_o^2} )</td>
</tr>
</tbody>
</table>

Table 4. Second Order 2 OTAs Implementation Filter Parameters.

<table>
<thead>
<tr>
<th>Circuit Type</th>
<th>Transfer Function</th>
<th>( w_o^* )</th>
<th>Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lowpass, ( w_o ) Adjustable, Q Fixed, Fig 5(a)</td>
<td>( \frac{(G_{m1} G_{m2}/C_1 C_2)}{s^2 + s(G_{m2}/C_2) + (G_{m1} G_{m2}/C_1 C_2)} )</td>
<td>( \frac{G_m}{\sqrt{C_1 C_2}} )</td>
<td>( \sqrt{C_2} )</td>
</tr>
<tr>
<td>Highpass, ( w_o ) Adjustable, Q Fixed, Fig 5(b)</td>
<td>( \frac{s^2}{s^2 + s(G_{m2}/C_2) + (G_{m1} G_{m2}/C_1 C_2)} )</td>
<td>( \frac{G_m}{\sqrt{C_1 C_2}} )</td>
<td>( \sqrt{C_2} )</td>
</tr>
<tr>
<td>Bandpass, ( w_o ) Adjustable, Q Fixed, Fig 5(c)</td>
<td>( \frac{s(G_{m2}/C_2)}{s^2 + s(G_{m2}/C_2) + (G_{m1} G_{m2}/C_1 C_2)} )</td>
<td>( \frac{G_m}{\sqrt{C_1 C_2}} )</td>
<td>( \sqrt{C_2} )</td>
</tr>
<tr>
<td>Bandreject, ( w_o ) Adjustable, Q Fixed, Fig 5(d)</td>
<td>( \frac{s^2 + (G_{m1} G_{m2}/C_1 C_2)}{s^2 + s(G_{m2}/C_2) + (G_{m1} G_{m2}/C_1 C_2)} )</td>
<td>( \frac{G_m}{\sqrt{C_1 C_2}} )</td>
<td>( \sqrt{C_2} )</td>
</tr>
</tbody>
</table>

* Assume \( G_m = G_{m1} = G_{m2} \)

The capacitances can be computed from any of the four transfer functions. The lowpass transfer function will be used for this purpose. The transfer function to be implemented is:

\[
L_2(s) = \frac{w^2}{s^2 + 0.61803 \cdot w \cdot s + w^2} = \frac{w_o^2}{s^2 + (w_o/Q)s + w_o^2}
\]
The capacitances are computed as follows:

\[ w_o^2 = w^2 \]

\[ w_o = w = \frac{G_m}{\sqrt{C_1 C_2}} \]

\[ C_1 C_2 = \left[ \frac{G_m}{w} \right]^2 \]

\[ \frac{w_o}{Q} = 0.61803 \times w \]

\[ Q = \frac{w_o}{0.61803 \times w} = \frac{w}{0.61803 \times w} = 1.61803 \]

\[ Q = \sqrt{\frac{C_2}{C_1}} \]

\[ C_2 = Q^2 C_1 \]

\[ Q^2 C_1^2 = \left[ \frac{G_m}{w} \right]^2 = C^2 \]

\[ C = \frac{G_m}{w} = \frac{85.44E - 6}{6.2832E + 5} = 135.98E - 12 = 135.98pF \]

\[ C_1 = \frac{1}{Q} C = \frac{1}{1.61803} 135.98E - 12 = 84.04E - 12 \]

\[ C_2 = Q^2 C_1 = QC = 1.61803(135.98E - 12) = 220.02E - 12 \]


Adjustable Q Implementation

(a) LP(s)

(b) HP(s)

(c) BP(s)

(d) BR(s)

Figure 6. Second Order Filters Implementation with adjustable Q and 3 OTAs each.

Figure 6 shows the second order filter implementation using 3 OTAs. The implementation has adjustable Q, its value is determined by the third OTA. The transfer function derivation will be demonstrated for the lowpass circuit. In Figure 6(a):
\[ I_1 = G_{m1}(V_i - V_o) \quad I_2 = G_{m2}V_1 \]

\[ V_1 = \frac{I_1}{sC_1} = \frac{G_{m1}}{sC_1}(V_i - V_o) \]

\[ I_2 = \frac{G_{m1}G_{m2}}{sC_1}(V_i - V_o) \]

\[ I_3 = -G_{m3}V_o \]

\[ V_o = \frac{1}{sC_2}[I_2 + I_3] = \frac{1}{sC_2}\left[ \frac{G_{m1}G_{m2}}{sC_2}(V_i - V_o) - G_{m3}V_o \right] \]

\[ \frac{V_o}{V_i} = \frac{G_{m1}G_{m2}}{s^2C_1C_2 + sC_1G_{m3} + G_{m1}G_{m2}} = \frac{G_{m1}G_{m2}}{C_1C_2} \]

The \( w_o \) and \( Q \) can be derived by comparing this transfer function with the standard lowpass transfer function given in Table.

\[ w_o = \sqrt{\frac{G_{m1}G_{m2}}{C_1C_2}} = \frac{G_m}{C}, \text{ if } G_m = G_{m1} = G_{m2} \text{ and } C = C_1 = C_2 \]

\[ \frac{w_o}{Q} = \frac{G_{m3}}{C_2} \]

Solving for \( Q \)

\[ Q = \frac{w_oC}{G_{m3}G_{m3}} = \frac{G_m}{G_{m3}} \]

NOTE for a given capacitor value, \( Q \) and \( w_o \) can be adjusted independently of each other. \( Q \) can be adjusted by \( G_{m3} \) and \( w_o \) by \( G_m \).

The expressions for the other circuits in Figure 6 can similarly be derived. The results are summarized in Table.
Table 5. Second Order 3 OTAs Implementation Filter Parameters.

<table>
<thead>
<tr>
<th>Circuit Type</th>
<th>Transfer Function</th>
<th>( w_o )</th>
<th>Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lowpass ( w_o ), Adjustable Q Adjustable, Fig 6(a)</td>
<td>( \frac{(G_{m1}G_{m2}/C_1C_2)}{s^2 + s(G_{m3}/C_2) + (G_{m1}G_{m2}/C_1C_2)} )</td>
<td>( \frac{G_m}{C} )</td>
<td>( \frac{G_m}{G_{m3}} )</td>
</tr>
<tr>
<td>Highpass ( w_o ), Adjustable Q Adjustable, Fig 6(b)</td>
<td>( \frac{s^2}{s^2 + s(G_{m3}/C_2) + (G_{m1}G_{m2}/C_1C_2)} )</td>
<td>( \frac{G_m}{C} )</td>
<td>( \frac{G_m}{G_{m3}} )</td>
</tr>
<tr>
<td>Bandpass ( w_o ), Adjustable Q Adjustable, Fig 6(c)</td>
<td>( \frac{s(G_{m3}/C_2)}{s^2 + s(G_{m3}/C_2) + (G_{m1}G_{m2}/C_1C_2)} )</td>
<td>( \frac{G_m}{C} )</td>
<td>( \frac{G_m}{G_{m3}} )</td>
</tr>
<tr>
<td>Bandreject ( w_o ), Adjustable Q Adjustable, Fig 6(d)</td>
<td>( \frac{s^2 + (G_{m1}G_{m2}/C_1C_2)}{s^2 + s(G_{m3}/C_2) + (G_{m1}G_{m2}/C_1C_2)} )</td>
<td>( \frac{G_m}{C} )</td>
<td>( \frac{G_m}{G_{m3}} )</td>
</tr>
</tbody>
</table>

* Assume \( G_m=G_{m1}=G_{m2}; C=C_1=C_2 * \)

The 3 OTAs implementation will illustrated using the same transfer function \( L_2(s) \) as in the 2 OTAs implementation.

\[
\begin{align*}
{w_o}^2 &= w^2 \\
{w_o} &= w \\
C &= \frac{G_m}{w_o} = \frac{85.44E-6}{6.2832E+5} = 135.98\text{pF} \\
\frac{w_o}{Q} &= 0.61803 \times w \\
Q &= \frac{w_o}{0.61803 \times w} = \frac{w}{0.61803 \times w} = 1.61803 \\
G_{m3} &= \frac{G_m}{Q} = \frac{85.44E-6}{1.61803} = 52.8E-6
\end{align*}
\]

As in the first order filter, universal filter implementation of second order filter in Figure 5 and figure 6 are shown in Figure 7(a) and 7(c) respectively. The corresponding programming are shown in Figure 7(b) and 7(d) respectively.
Figure 7. Universal Second Order Filter Implementations.
4.1 Second Order Filters Simulation

Second Order Lowpass Filter Transfer Function

*Filename= "LP2.cir"
*Second order lowpass filter
.PARAM w=6.2832e+5
Vin 1 0 DC 0V AC 1V

* third biquad
R1 1 0 1E+20
E2 2 0 LAPLACE \{V(1)\}=\frac{(w+w)/(s+s+0.61803*w*s+w*w)}

*output
Rout 2 0 1k
* Analysis
.AC DEC 100 1Hz 100MegHz
.PROBE
.END

Second Order Highpass Filter Transfer Function

*Filename= "HP2.cir"
*Second order highpass filter
.PARAM w=6.2832e+5
Vin 1 0 DC 0V AC 1V

* third biquad
R1 1 0 1E+20
E2 2 0 LAPLACE \{V(1)\}=\frac{(s*s)/(s*s+0.61803*w*s+w*w)}

*output
Rout 2 0 1k
* Analysis
.AC DEC 100 1Hz 100MegHz
.PROBE
.END

Second Order Bandpass Filter Transfer Function

*Filename="BP2.cir"
*Second order bandpass filter
.PARAM w=6.2832e+5
Vin 1 0 DC 0V AC 1V

* third biquad
R1 1 0 1E+20
E2 2 0 LAPLACE \{V(1)\}=\frac{(0.61803*w*s)/(s*s+0.61803*w*s+w*w)}

*output
Rout 2 0 1k
* Analysis
.AC DEC 100 1Hz 100MegHz
.PROBE
.END

Second Order Bandreject Filter Transfer Function

*Filename="BR2.cir"
*Second order bandreject filter
.PARAM w=6.2832e+5
Vin 1 0 DC 0V AC 1V

* third biquad
R1 1 0 1E+20
E2 2 0 LAPLACE {V(1)}={(s*s+w*w)/(s*s+0.61803*w*s+w*w)}

*output
Rout 2 0 1k
* Analysis
.AC DEC 100 1Hz 100MegHz
.PROBE
.END

Ideal Second Order Lowpass Filter Response
Ideal Second Order Highpass Filter Response

Ideal Second Order Bandpass Filter Response
Ideal Second Order Bandreject Response

Universal Second Order Filter Using 2 Ideal OTAs per Filter

*Filename = "univ2_i2.cir", see Figure 7a
*Biquad 2nd order filter. fo=100K

Vin 1 0 DC 0V AC 1V
*LP
*Xs2flt 1 0 0 2 S2FLT
*HP
*Xs2flt 0 0 1 2 S2FLT
*BP

*Xs2flt 0 1 0 2 S2FLT
*BR
Xs2flt 1 0 1 2 S2FLT

R2 2 0 1E+20
.SUBCKT S2FLT v1 v2 v3 v0
.PARAM C=135.98pF
.PARAM Q=1.61803
Xota1 v1 v0 n2 WSOTA
Xota2 n2 v0 v0 WSOTA
C1 n2 v2 (C/Q) IC=0V
C2 v0 v3 (Q*C) IC=0V
Rn2 n2 0 1E+20
. ENDS

.SUBCKT WSOTA in+ in- out
G1 0 out in+ in- 85.44U
. ENDS
Universal Second Order Filter Using Ideal 3 OTAs per Filter

*Filename = "univ2_i3.cir", see Figure 7c
*Biquad 2nd order filter. fo=100K

Vin 1 0 DC 0V AC 1V
*LP
*Xs3flt 1 0 0 2 S3FLT
*HP
*Xs3flt 0 1 0 2 S3FLT
*BP
*Xs3flt 0 0 1 2 S3FLT
*BR
Xs3flt 1 1 0 2 S3FLT

R2 2 0 1E+20

.SUBCKT S3FLT v1 v2 v3 v0
.PARAM C=135.98pF
Xota1 v1 v0 n2 WSOTA
Xota2 n2 0 v0 WSOTA
Xota3 v3 v0 v0 WSOTA1
C1 n2 0 {C} IC=0V
C2 v0 v2 {C} IC=0V
Rn2 n2 0 1E+20
.ENDED

.SUBCKT WSOTA in+ in- out
G1 0 out in+ in- 85.44U
.ENDED

.SUBCKT WSOTA1 in+ in- out
G1 0 out in+ in- 52.8U
.ENDED

* Analysis
.AC DEC 100 1Hz 100MegHz
.PROBE
.END
Second Order Lowpass Filter Response

Second Order Highpass Filter Response
Second Order Bandpass Response

Second Order Bandreject Response
5. Butterworth Filters Implementation

Table 6. Butterworth D(sn) (where H(sn)=K/D(sn); K=1)

<table>
<thead>
<tr>
<th>N</th>
<th>D(sn)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>s_n + 1</td>
</tr>
<tr>
<td>2</td>
<td>s_n^2 + √2s_n + 1</td>
</tr>
<tr>
<td>3</td>
<td>(s_n + 1)(s_n^2 + s_n + 1)</td>
</tr>
<tr>
<td>4</td>
<td>(s_n^2 + 0.76537s_n + 1)(s_n^2 + 1.84776s_n + 1)</td>
</tr>
<tr>
<td>5</td>
<td>(s_n + 1)(s_n^2 + 0.61803s_n + 1)(s_n^2 + 1.61803s_n + 1)</td>
</tr>
</tbody>
</table>

The fifth order normalized Butterworth low pass filter is obtained from the above table:

\[ B_5(s_n) = \frac{1}{(s_n + 1)(s_n^2 + 1.61803s_n + 1)(s_n^2 + 0.61803s_n + 1)} \]

To scale the transfer function to the desired frequency of operation, the transformation \( s_n = s/w \) is applied to the normalized transformation.

\[
B_5(s) = \left[ \frac{w}{s + w} \right] \left[ \frac{w^2}{s^2 + 1.61803w^*s + w^2} \right] \left[ \frac{w^2}{s^2 + 0.61803w^*s + w^2} \right]
\]

\[ = T_1(s)T_2(s)T_3(s) \]

This is a cascade design based on the simple tandem interconnection on N first-order and biquad stages. If the stages are non-interacting, the overall transfer function is the product of the individual stage transfer functions. Non-interacting means that at any stage, say the ith stage, the output impedance \( Z_{out(i)} \) is not loaded by the input impedance \( Z_{in(i+1)} \) of the succeeding stage, i.e., \( |Z_{out(i)}|<<|Z_{in(i+1)}| \). For op amp-based active filters, this is inherently satisfied, since op amp has high input impedance and low output impedance. But OTA based active filters, due to high input and output impedances require inter-stage buffering. These buffers are usually realized by unity-gain source follower stages. In addition, output buffering is required to drive external loads.

The capacitance and transconductance of each OTA will be computed for each cascaded filter.

For \( T_1(s) \):

\[ w_o = w \]

\[ C = \frac{G_m}{w_o} = \frac{85.44E-6}{6.2832E+5} = 135.98 \text{pF} \]

For \( T_2(s) \):
\[ w_o^2 = w^2 \]
\[ w_o = w \]
\[ C = \frac{G_m}{w_o} = \frac{85.44E-6}{6.2832E+5} = 135.98 \text{pF} \]
\[ \frac{w_o}{Q} = 1.61803 * w \]
\[ Q = \frac{w_o}{1.61803 * w} = \frac{w}{1.61803 * w} = 0.618 \]
\[ G_{m3} = \frac{G_m}{Q} = \frac{85.44E-6}{0.618} = 138.25E-6 \]

For \( T3(s) \):
\[ w_o^2 = w^2 \]
\[ w_o = w \]
\[ C = \frac{G_m}{w_o} = \frac{85.44E-6}{6.2832E+5} = 135.98 \text{pF} \]
\[ \frac{w_o}{Q} = 0.61803 * w \]
\[ Q = \frac{w_o}{0.61803 * w} = \frac{w}{0.61803 * w} = 1.618 \]
\[ G_{m3} = \frac{G_m}{Q} = \frac{85.44E-6}{1.61803} = 52.8E-6 \]

### 5.1 Fifth Order Butterworth Filter Simulation

**Fifth Order Butterworth Filter Transfer Function**

*Filename = "butter5.cir"
*Fifth order butterworth filter, fo=100k
.PARAM w=6.2832e+5
Vin 1 0 DC 0V AC 1V

*first biquad
R1 1 0 1E+20
E1 2 0 LAPLACE \{V(1)\} = \{(w)/(s+w)\}

* second biquad
R2 2 0 1E+20
E2 3 0 LAPLACE \{V(2)\} = \{(w*w)/(s*s+1.61803*w*s+w*w)\}

* third biquad
R3 3 0 1E+20
E3 4 0 LAPLACE \{V(3)\} = \{(w*w)/(s*s+0.61803*w*s+w*w)\}

*output

33
Fifth Order Butterworth Filter Implementation Using Ideal 3 OTAs per Filter

*Filename = "but5_i3.cir"
*Butterworth 5th order filter. fo=100K

Vin 1 0 DC 0V AC 1V
Xs1flt 1 0 2 S1FLT
Xbuf1 2 3 BUF
Xs2flt 3 0 0 4 S2FLT
Xbuf2 4 5 BUF
Xs3flt 5 0 0 6 S3FLT
R2 2 0 1E+20
R3 3 0 1E+20
R4 4 0 1E+20
R5 5 0 1E+20
R6 6 0 1E+20

.SUBCKT S1FLT v1 v2 v0
.PARAM C=135.98pF
Xota1 v1 v0 v0 WSOTA
C1 v0 v2 (C)
.ENDS

.SUBCKT S2FLT v1 v2 v3 v0
.PARAM C=135.98pF
Xota1 v1 v0 n2 WSOTA
Xota2 n2 0 v0 WSOTA
Xota3 v3 v0 v0 WSOTA2
C1 n2 0 {C} IC=0V
C2 v0 v2 (C) IC=0V
Rn2 n2 0 1E+20
.ENDS

.SUBCKT S3FLT v1 v2 v3 v0
.PARAM C=135.98pF
Xota1 v1 v0 n2 WSOTA
Xota2 n2 0 v0 WSOTA
Xota3 v3 v0 v0 WSOTA3
C1 n2 0 {C} IC=0V
C2 v0 v2 (C) IC=0V
Rn2 n2 0 1E+20
.ENDS

.SUBCKT WSOTA in+ in- out
G1 0 out in+ in- 85.44U
.ENDS

.SUBCKT WSOTA2 in+ in- out
G1 0 out in+ in- 138.25U
.ENDS

.SUBCKT WSOTA3 in+ in- out
G1 0 out in+ in- 52.8U
.ENDS

.SUBCKT BUF in out
EI out 0 in 0 1
.ENDS

* Analysis
.AC DEC 100 1Hz 100MegHz
.PROBE
.END

Ideal Fifth Order Butterworth Filter Response (in DB)

Ideal Fifth Order Butterworth Filter Response (in V)
Implemented Fifth Order Butterworth Response (in DB)

Implemented Fifth Order Butterworth Response (in V)
6. Chebyshev Filters Implementation

Chebyshev D(sn) for 1dB ripple (Where H(sn) = K/D(sn); K selected to yield unity DC gain)

<table>
<thead>
<tr>
<th>N</th>
<th>D(sn)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( s_n + 1.96523 )</td>
</tr>
<tr>
<td>2</td>
<td>( s_n^2 + 1.09773s_n + 1.10251 )</td>
</tr>
<tr>
<td>3</td>
<td>( (s_n + 0.49417)(s_n^2 + 0.49417s_n + 0.9942) )</td>
</tr>
<tr>
<td>4</td>
<td>( (s_n + 0.27907s_n + 0.9865)(s_n^2 + 0.67374s_n + 0.2794) )</td>
</tr>
<tr>
<td>5</td>
<td>( (s_n + 0.28949)(s_n^2 + 0.17892s_n + 0.98831)(s_n^2 + 0.46841s_n + 0.4293) )</td>
</tr>
</tbody>
</table>

The fifth order normalized Chebyshev low pass filter is obtained from the above table:

\[
\text{Ch5}(s_n) = \frac{(0.28949)(0.98831)(0.4293)}{(s_n + 0.28949)(s_n^2 + 0.17892s_n + 0.98831)(s_n^2 + 0.46841s_n + 0.4293)}
\]

To scale the transfer function to the desired frequency of operation, the transformation \( s_n = \frac{s}{w} \) is applied to the normalized transformation.

\[
\text{Ch5}(s) = \frac{0.28949*w}{s + 0.28949*w} \times \frac{0.98831*w^2}{s^2 + 0.17892*w*s + 0.98831*w^2} \times \frac{0.4293*w^2}{s^2 + 0.46841*w*s + 0.4293*w^2}
\]

\[
= \text{T1}(s)\text{T2}(s)\text{T3}(s)
\]

The capacitance and transconductance of each OTA will be computed for each cascaded filter.

For T1(s):

\[
w_o = 0.28949 * w
\]

\[
C = \frac{G_m}{w_o} = \frac{85.44E-6}{0.28949*6.2832E+5} = 469.73\text{pF}
\]

For T2(s):
\[ w_0^2 = 0.98831 \times w^2 \]
\[ w_0 = 0.994 \times w \]
\[ C = \frac{G_m}{w_0} = \frac{85.44E-6}{0.994 \times 6.2832E+5} = 136.8pF \]
\[ \frac{w_0}{Q} = 0.17892 \times w \]
\[ Q = \frac{w_0}{0.17892 \times w} = \frac{0.994 \times w}{0.17892 \times w} = 5.56 \]
\[ G_{m3} = \frac{G_m}{Q} = \frac{85.44E-6}{5.56} = 15.37E-6 \]

For T3(s):
\[ w_0^2 = 0.4293 \times w^2 \]
\[ w_0 = 0.6552 \times w \]
\[ C = \frac{G_m}{w_0} = \frac{85.44E-6}{0.6552 \times 6.2832E+5} = 207.5pF \]
\[ \frac{w_0}{Q} = 0.46841 \times w \]
\[ Q = \frac{w_0}{0.46841 \times w} = \frac{0.6552 \times w}{0.46841 \times w} = 1.4 \]
\[ G_{m3} = \frac{G_m}{Q} = \frac{85.44E-6}{1.4} = 61.03E-6 \]

### 6.1 Fifth Order Chebyshev Filter Simulation

**Fifth Order Chebyshev Filter Transfer Function**

*Filename= cheby5.cir*
*Cheby5 Fifth order chebyshev filter*
*First order filter*
*.PARAM w=6.2832e+5*
*Vin 1 0 DC 0V AC 1V*

*first biquad*
R1 1 0 1E+20
E1 2 0 LAPLACE \{V(1)\} = \{(0.28949\times w) / (s+0.28949\times w)\}

* second biquad*
R2 2 0 1E+20
E2 3 0 LAPLACE \{V(2)\} = \{(0.4293\times w\times w) / (s\times s+0.4684\times w\times s+0.4293\times w\times w)\}

* third biquad*
Fifth Order Chebyshev Filter Implementation Using Ideal 3 OTAs per Filter

*Filename = "chb5_i3.cir"
*Chebyshev 5th order filter. fo=100K

Vin 1 0 DC 0V AC 1V

.XSUBCKT S1FLT v1 v2 v0
.PARAM C=469.73pF
.XOTA1 v1 v0 v0 WSOTA
.C1 v0 v2 (C)
.ENDS

.XSUBCKT S2FLT v1 v2 v3 v0
.PARAM C=136.8pF
.XOTA1 v1 v0 n2 WSOTA
.XOTA2 n2 0 v0 WSOTA
.XOTA3 v3 v0 v0 WSOTA2
.C1 n2 0 (C) IC=0V
.C2 v0 v2 (C) IC=0V
.Rn2 n2 0 1E+20
.ENDS

.XSUBCKT S3FLT v1 v2 v3 v0
.PARAM C=207.5pF
.XOTA1 v1 v0 n2 WSOTA
.XOTA2 n2 0 v0 WSOTA
.XOTA3 v3 v0 v0 WSOTA3
.C1 n2 0 (C) IC=0V
.C2 v0 v2 (C) IC=0V
.Rn2 n2 0 1E+20
.ENDS

.XSUBCKT WSOTA in+ in- out
.G1 0 out in+ in- 85.44U
.ENDS

.XSUBCKT WSOTA2 in+ in- out
.G1 0 out in+ in- 15.37U
.ENDS

.XSUBCKT WSOTA3 in+ in- out
.G1 0 out in+ in- 61.03U
.ENDS
* Analysis
.AC DEC 100 1Hz 100MegHz
.PROBE
.END

Ideal Fifth Order Chebyshev Filter Response (in DB)

Ideal Fifth Order Chebyshev Filter Response (in V)
Implemented Fifth Order Chebyshev Filter Response (in DB)

![Diagram of a general biquad implementation](image)

Figure 8. General Second Order Filter Implementation.

Figure 8 shows the single-ended OTA implementation of general second order filter. The transfer function is derived as follows:
\[ I_1 = -G_{m1} V_o \]
\[ I_2 = G_{m2} V_i \]
\[ I_3 = -G_{m3} V_o \]
\[ I_4 = G_{m4} V_i \]
\[ I_5 = G_{m5} V_i \]

\[ I_{c3} = s C_3 (V_i - V_o) \]
\[ I_{c1} = I_1 + I_4 = -G_{m1} V_o + G_{m4} V_i \]

\[ V_i = \frac{1}{s C_1} I_{c1} = \frac{1}{s C_1} (-G_{m1} V_o + G_{m4} V_i) \]

\[ I_{c2} = I_2 + I_3 + I_5 + I_{c3} = G_{m2} V_i - G_{m3} V_o + G_{m5} V_i + s C_3 (V_i - V_o) = s C_2 V_o \]

\[
\frac{V_o}{V_i} = \frac{s^2 C_1 C_3 + s C_1 G_{m5} + G_{m2} G_{m4}}{s^2 C_1 (C_2 + C_3) + s C_1 G_{m3} + G_{m1} G_{m4}}
\]

\[
\frac{V_o}{V_i} = \frac{s^2 \left( \frac{C_3}{C_2 + C_3} \right) + s \left( \frac{G_{m5}}{C_2 + C_3} \right) + \left( \frac{G_{m2} G_{m4}}{C_1 (C_2 + C_3)} \right)}{s^2 + s \left( \frac{G_{m3}}{C_2 + C_3} \right) + \left( \frac{G_{m1} G_{m4}}{C_1 (C_2 + C_3)} \right)} = \frac{a_2 s^2 + a_1 s + a_0}{s^2 + \left( \frac{w_o}{Q} \right) + w_o^2}
\]

It will be shown that general implementation of 5 OTAs reduces to 3 or 2 OTAs for the standard filters such as lowpass filter.

First it will be shown for lowpass transfer function general implementation reduces to 4 OTAs. Comparing the two transfer functions,

\[
\frac{V_o}{V_i} = \frac{s^2 \left( \frac{C_3}{C_2 + C_3} \right) + s \left( \frac{G_{m5}}{C_2 + C_3} \right) + \left( \frac{G_{m2} G_{m4}}{C_1 (C_2 + C_3)} \right)}{s^2 + s \left( \frac{G_{m3}}{C_2 + C_3} \right) + \left( \frac{G_{m1} G_{m4}}{C_1 (C_2 + C_3)} \right)} = \frac{w_o^2}{s^2 + \left( \frac{w_o}{Q} \right) + w_o^2}
\]

Equating the coefficients,
\[
\frac{C_3}{C_2 + C_3} = 0 \Rightarrow C_3 = 0 \Rightarrow \text{open}
\]

\[
\frac{G_{m5}}{C_2 + C_3} = 0 \Rightarrow G_{m5} = 0
\]

That means that both capacitor C3 and Gm5 can be deleted, resulting in the circuit shown in Figure 9(a). The 4 OTAs implementation can be reduced to 3 OTAs if Gm1=Gm4. This is shown below:

![Circuit Diagrams](image)

Figure 9. General Biquad Lowpass Filter Transformation: (a) 4 OTAs, (b) 3 OTAs, and (c) 2 OTAs.
\[ I_1 = -G_{m1}V_o \]
\[ I_4 = G_{m4}V_i \]
\[ I_{C1} = I_1 + I_4 = -G_{m1}V_o + G_{m4}V_i = G_{m1}(V_i - V_o); G_{m1} = G_{m4} \]

That is, \( G_{m1} \) and \( G_{m4} \) can be combined as shown in Figure 9(b).

\[ I_{C1} = I_1 = G_{m1}(V_i - V_o) \]

The 3 OTAs can be reduced to 2 OTAs if \( G_{m2} = G_{m3} \). This is shown as follows:

\[ I_2 = G_{m2}V_i \]
\[ I_3 = -G_{m3}V_o \]
\[ I_{C2} = I_2 + I_3 = G_{m2}V_i - G_{m3}V_o = G_{m2}(V_i - V_o); G_{m2} = G_{m3} \]

That is, \( G_{m2} \) and \( G_{m3} \) can be combined as shown in Figure 9(c).

\[ I_{C2} = I_2 = G_{m2}(V_i - V_o) \]

Figure 10. Differential OTA Implementation of General second-order filter.