Chapter 7: Pulse Modulation

- Basic concepts
  - Modulation: a process by which a property of a parameter of a signal $\phi(t)$ is varied in proportional to a second (given) signal $f(t)$. We use modulation technique to alter signals in time and frequency to accomplish desired objectives.
  - Analog or continuous-wave (CW) modulation: $f(t)$ is used to vary a parameter of a high-frequency sinusoidal signal (AM, FM & PM): $\phi(t) = a(t)\cos\theta(t) = a(t)\cos\left[\int_0^t \omega(\tau)d\tau + \gamma(t)\right]$.
  - Sampling theorem: a band-limited analog signal can be reconstructed completely from a set of uniformly spaced discrete samples in time.
  - Analog pulse modulation: the discrete samples of $f(t)$ are used to vary a parameter of a periodic pulse signal (PAM, PWM & PPM): $\phi(t) = a(t)\,rect_{T(t)}\left[t/\tau(t)\right]$, where $T(t) \le (2f_m)^{-1}$.

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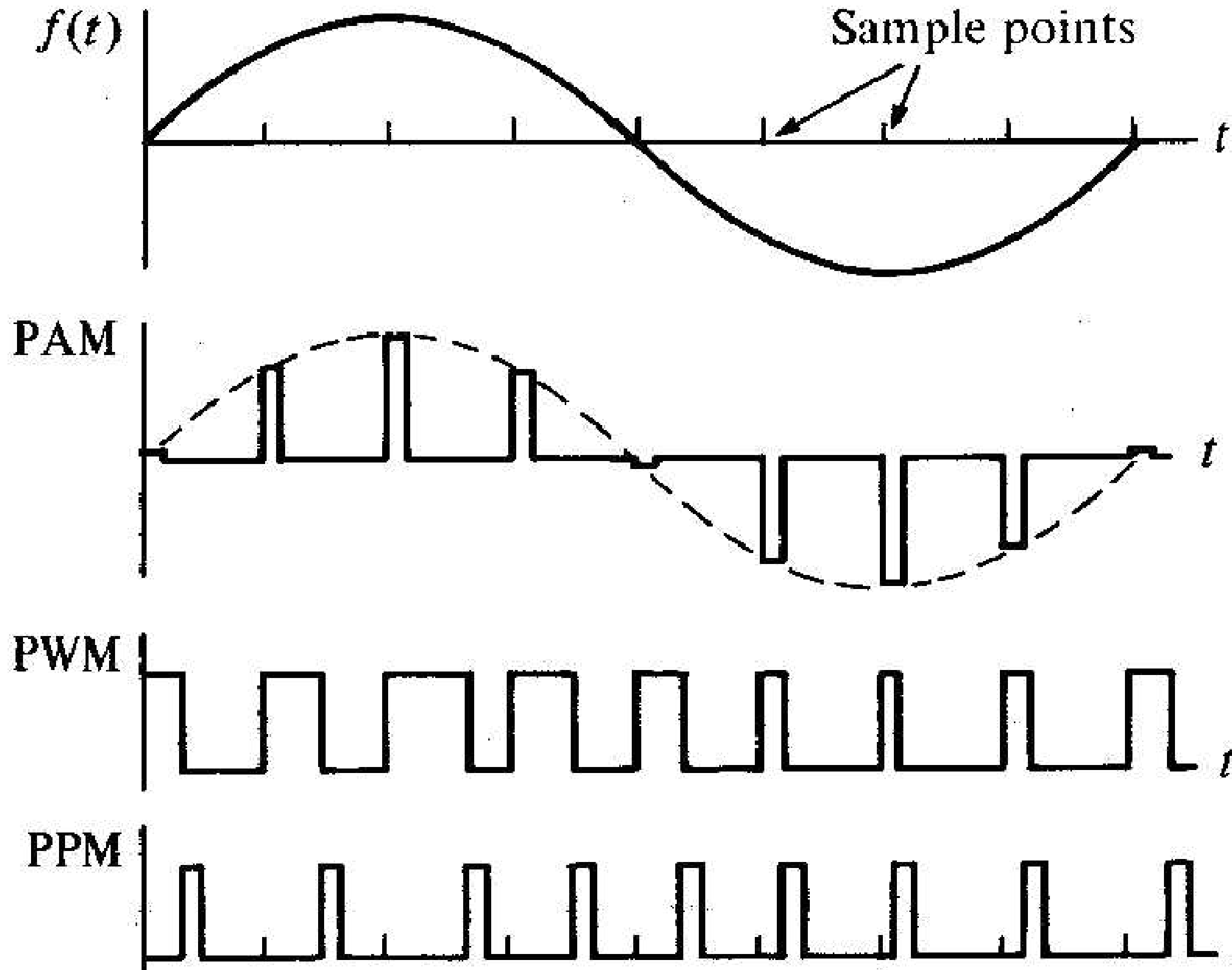
- Analog pulse modulation (graphic interpretation)

Modulating Signal

Pulse-Amplitude Modulation (PAM)

Pulse-Width Modulation (PWM)

Pulse-Position Modulation (PPM)



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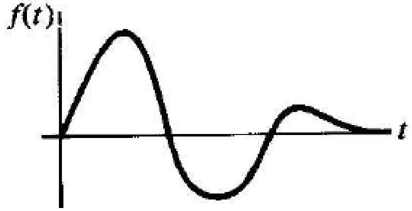
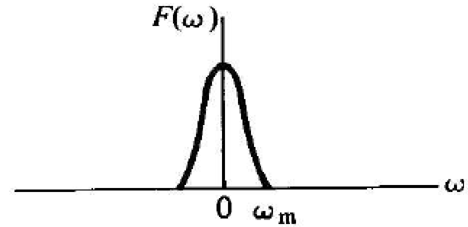
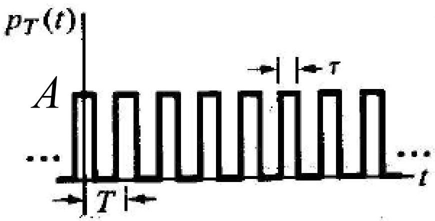
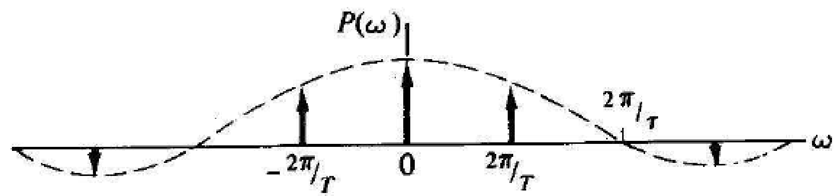
- Natural sampling of a band-limited signal (Chapter 3)





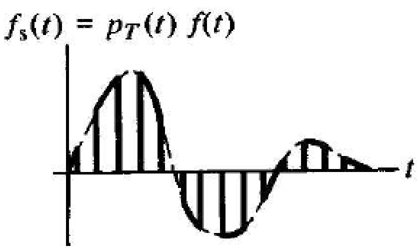
Nyquist sampling frequency (rate): $2f_m$.

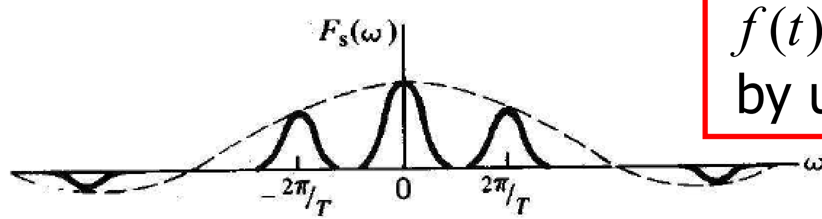




$$Sa(x) = \frac{\sin x}{x}$$

$$P(\omega) = \frac{2\pi\, A\tau}{T} \sum_{n=-\infty}^{\infty} Sa\left(\frac{n\pi\tau}{T}\right) \delta\left(\omega - \frac{2n\pi}{T}\right)$$





$f(t)$ can be recovered by using an ideal LPF.

Pulse tops varies with $f(t)$ at the sample points.

$$F_S(\omega) = \frac{1}{2\pi} F(\omega) * P(\omega) = \frac{A\tau}{T} \sum_{n=-\infty}^{\infty} Sa\left(\frac{n\pi\tau}{T}\right) F\left(\omega - \frac{2n\pi}{T}\right)$$

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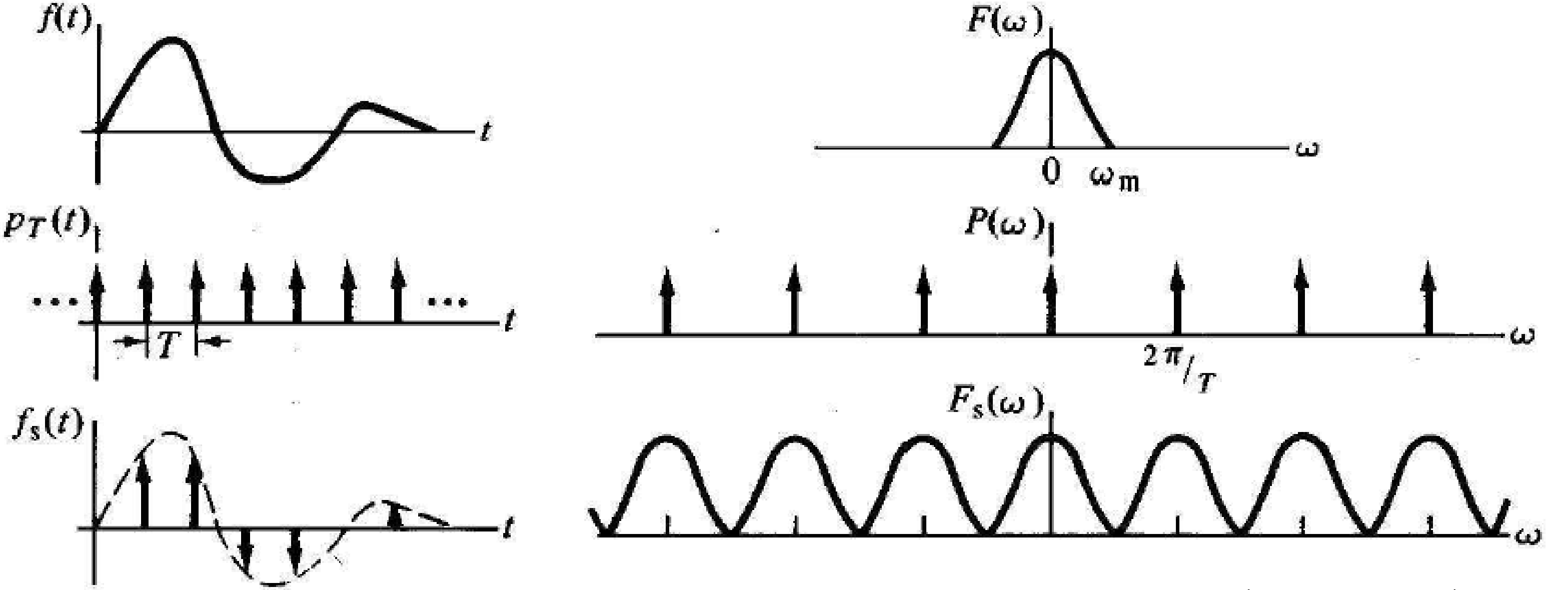
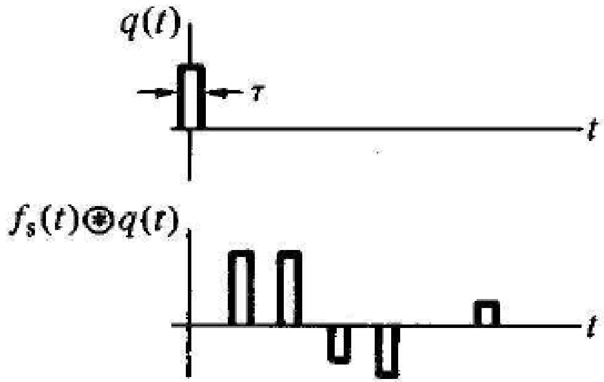
- Pulse-amplitude modulation (PAM)
  - Let $A\tau = 1$ & $\tau \to 0$, rectangular pulse becomes impulse.



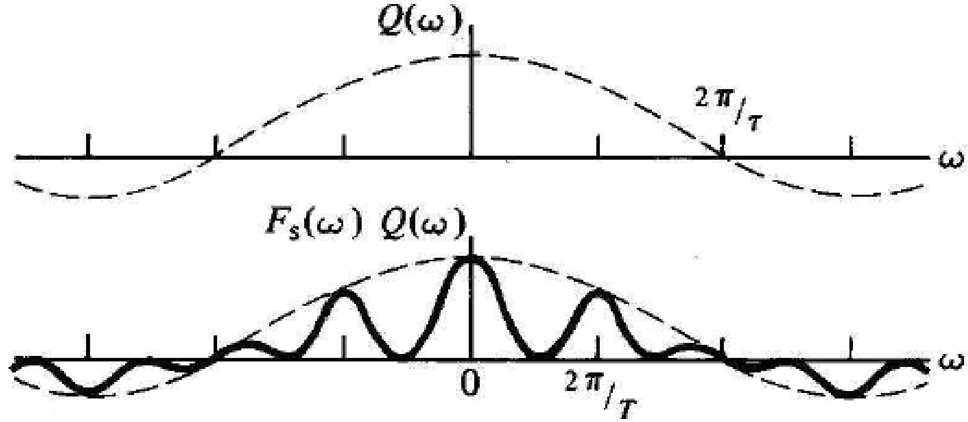
$$p_T(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT) \qquad P(\omega) = \frac{2\pi}{T} \sum_{n=-\infty}^{\infty} \delta\left(\omega - \frac{2n\pi}{T}\right)$$

$$f_S(t) = \sum_{n=-\infty}^{\infty} f(nT)\,\delta(t - nT) \qquad F_S(\omega) = \frac{1}{T} \sum_{n=-\infty}^{\infty} F\left(\omega - \frac{2n\pi}{T}\right)$$

- Pulse-amplitude modulation (PAM) (continued)



Pulse tops are flat.



Due to frequency distortion, $f(t)$ cannot be exactly recovered by simply a LPF.

$$q(t) = rect(t/\tau)$$

$$\phi_{PAM}(t) = f_S(t) * q(t)$$

$$= \sum_{n=-\infty}^{\infty} f(nT)\,\delta(t - nT) * q(t)$$

$$= \sum_{n=-\infty}^{\infty} f(nT)\,q(t - nT)$$

$$Q(\omega) = \tau\, Sa(\omega\tau/2)$$

$$\Phi_{PAM}(\omega) = F_S(\omega)\,Q(\omega)$$

$$= \frac{\tau}{T} \sum_{n=-\infty}^{\infty} F\left(\omega - \frac{2n\pi}{T}\right) Sa\left(\frac{\omega\tau}{2}\right)$$

Chapter 7: Pulse Modulation

- Demodulation of PAM signals
  - Method 1: sampling

    (1) sample the PAM waveform with a train of very narrow pulses;

    (2) use a low-pass filter to smooth the result.
  - Requirement of Method 1:

    (1) the sampling pulse train must be synchronized with the incoming PAM signal;

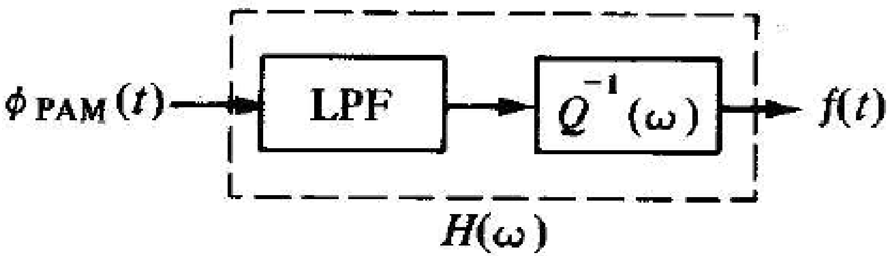
    (2) the sampling pulse should be very narrow (ideally impulse).
  - Method 2: filtering

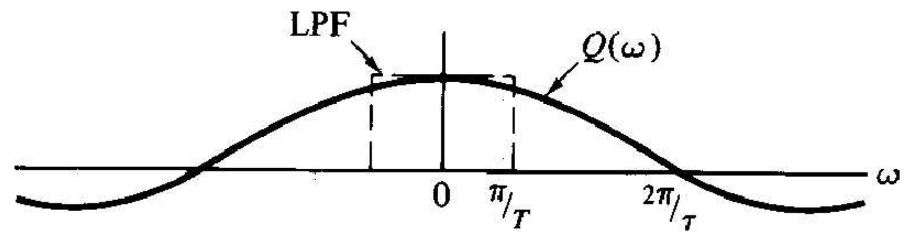
    (1) filter out the main-lobe by a LPF (cutoff frequency= f_m);

    (2) use an equalizer to correct the distortion and then recover $f(t)$.
    - Equalizer can correct the frequency response of a system for a known distortion which, however, can hardly be controlled.

Chapter 7: Pulse Modulation

- Demodulation of PAM signals (continued)
  - Method 2: filtering (graphic interpretation)

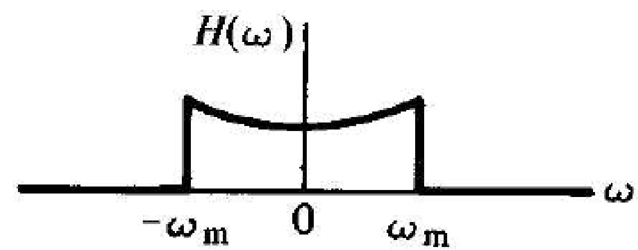




Assume $f_m = 1/(2T)$.

  - Requirement of Method 2:

(1) $$H(\omega) = \begin{cases} Q^{-1}(\omega), & |\omega| < \omega_m; \\ 0, & elsewhere. \end{cases}$$



(2) the ratio τ/T is a measure of the flatness of $Q(\omega)$ and $Q^{-1}(\omega)$ within the bandwidth of the low pass filter. In practice, the equalization for PAM can usually be neglected when $\tau/T \le 0.1$.

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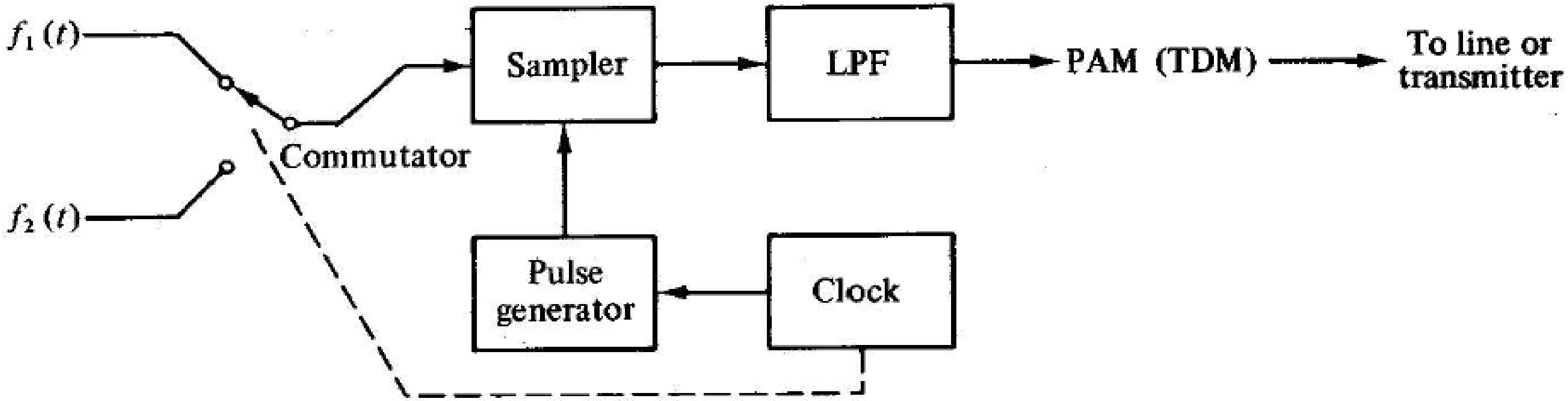
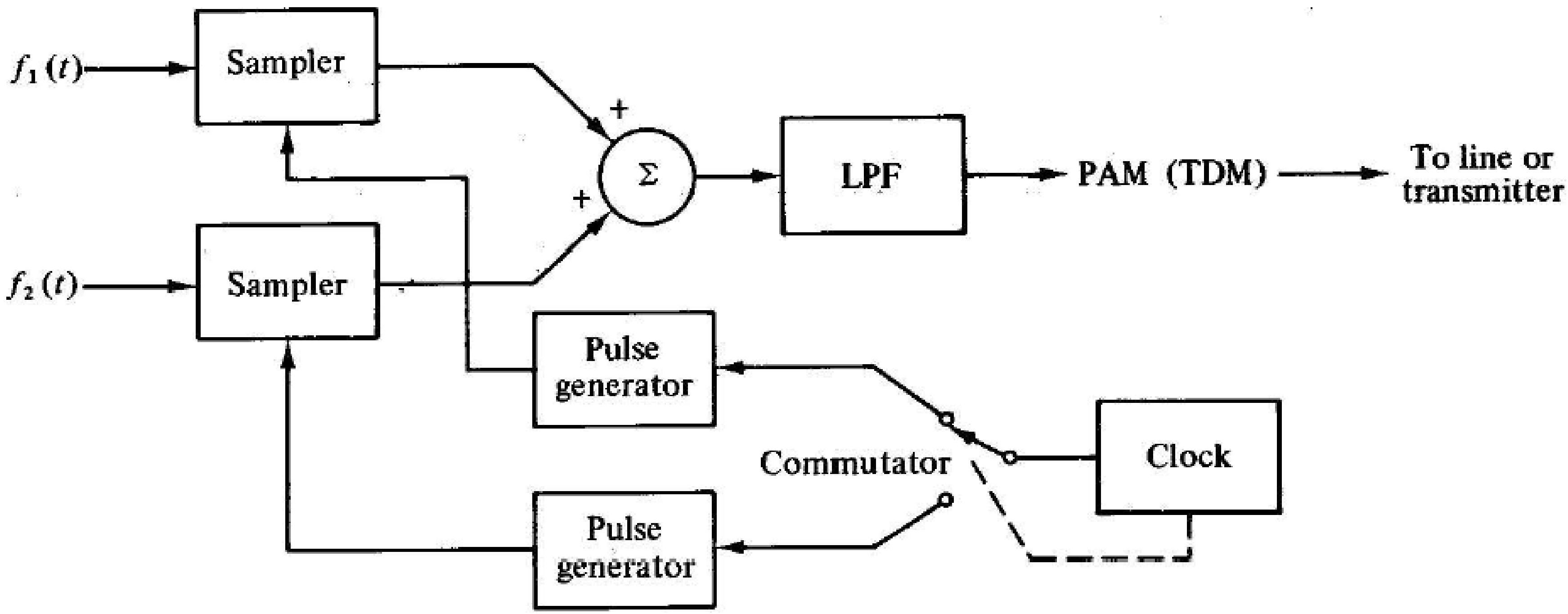
- Time-division multiplex (TDM)
  - Time-division multiplexing is the method of combining several sampled signals in a definite time sequence.
  - Commutator determines the synchronization and sequence of the channels (signals) to be sampled.
  - Time multiplexing of two PAM signals
    - Method 1: the commutator handles the analog signals before sampling



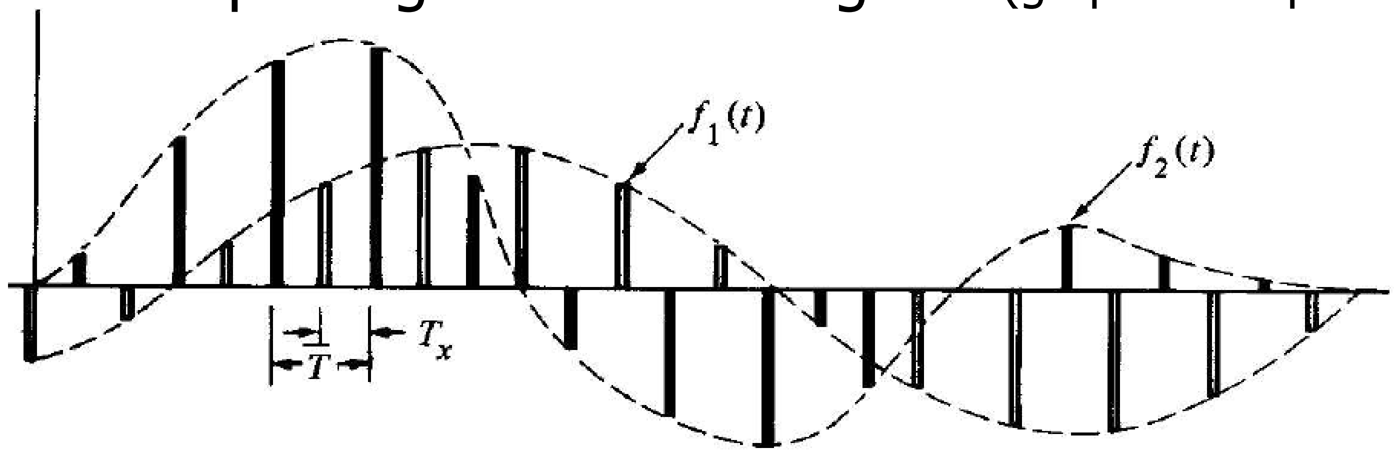
Chapter 7: Pulse Modulation

- Time-division multiplex (TDM) (continued)
  - Time multiplexing of two PAM signals (continued)
    - Method 2: the commutator handles the sampler control pulses



    - Method 2 is often preferred because it lends itself to digital logic circuitry even though it does require more samplers.

- Time-division multiplex (TDM) (continued)
  - Time multiplexing of two PAM signals (graphic interpretation)



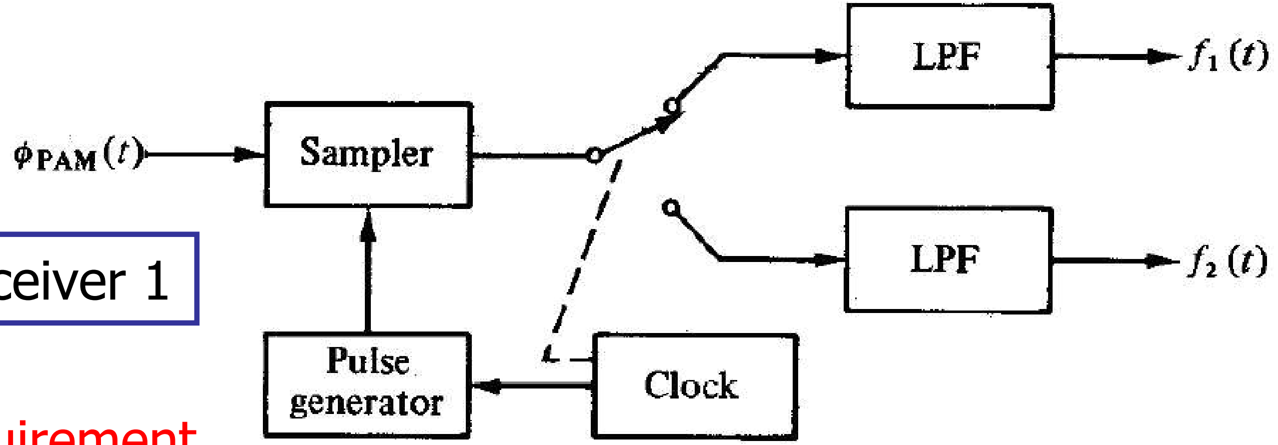
    - Assume all input signals are low-pass and band-limited to f_m.
    - Let T be the sampling period for one signal [$T \le (2f_m)^{-1}$].
    - Let n be the number of input PAM signals. Here, $n = 2$.
    - Let T_x be the time spacing between adjacent samples in the time-multiplexed signal waveform [$T_x = T/n$].
    - The bandwidth of time-multiplexed signal is very wide (infinite).

- Time-division multiplex (TDM) (continued)
  - Necessary bandwidth for TDM signal transmission
    - As only the amplitude information is important here, the absolute minimum bandwidth required such that the information in each sampled channel remains independent of that in the other channels is given by $B_x \geq \dfrac{1}{2T_x} = \dfrac{n}{2T} \geq nf_m$.
    - When all input signals have the same bandwidth f_m, PAM is just as efficient in conserving bandwidth as SSB.
    - The equality sign in the inequality refers to the case in which impulse sampling and ideal filtering are used. Because neither of these conditions holds in practice, the requirement on the bandwidth must be relaxed somewhat.
    - The use of this bound will not yield pulse shapes that closely resemble the pulses generated in the samplers. Pulse shape recognition requires additional bandwidth.

- Time-division multiplex (TDM) (continued)
  - Receivers for time-multiplexed PAM signals
    - Procedure

      (1) The composite time-multiplexed and filtered waveform is re-sampled and separated into the appropriate channels.
      (2) The normal sampling conditions now apply and the analog reconstruction of the signals can be obtained by LPF.

Receiver 1



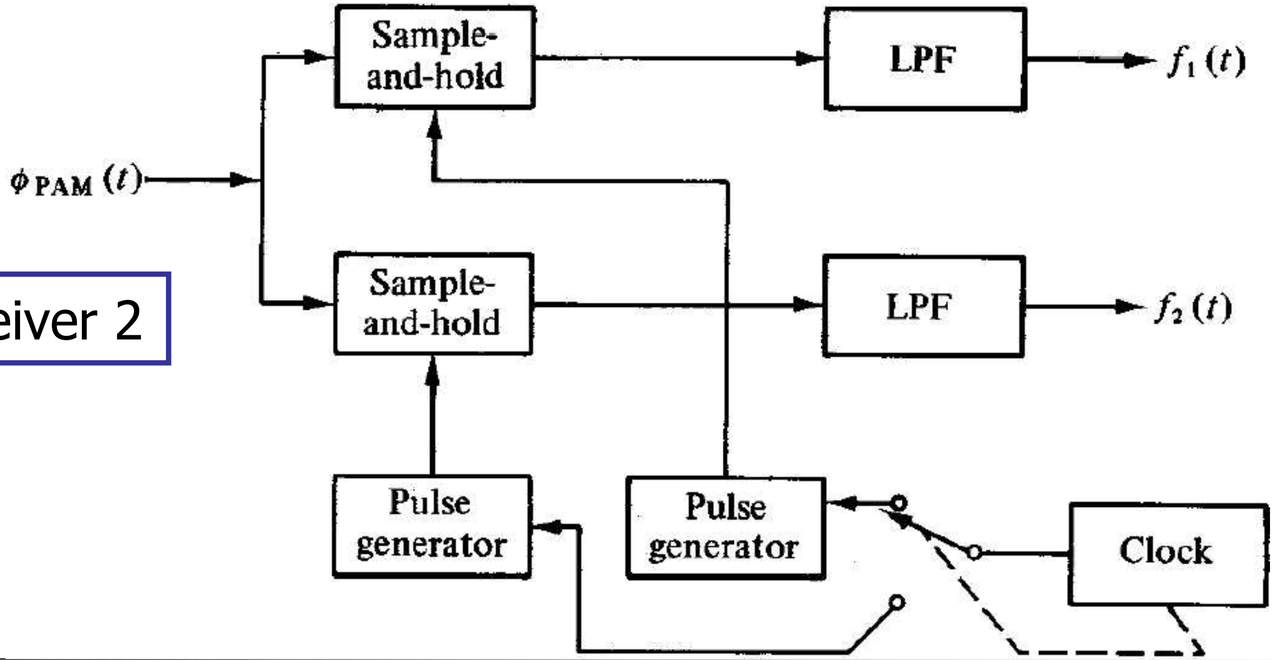
    - Requirement

      (1) Synchronization of the clock and the commutator in the time-multiplex receiver is necessary.

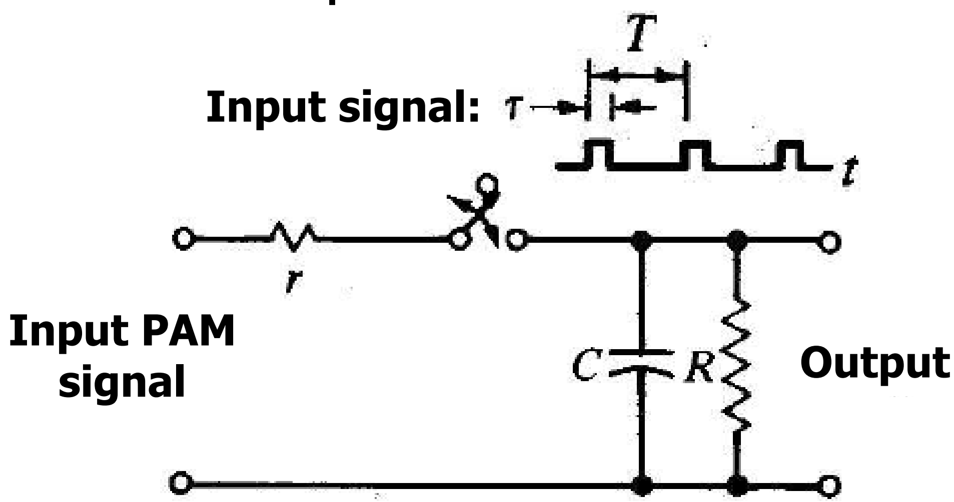
- Time-division multiplex (TDM) (continued)
  - Receivers for time-multiplexed PAM signals (requirement)

    (2) When a large number of PAM signals are time-multiplexed together, the width of the sampling pulses must be made very narrow. As a result, the final output signal power is small so that an amplifier or a sample-and-hold circuit is needed.

Receiver 2

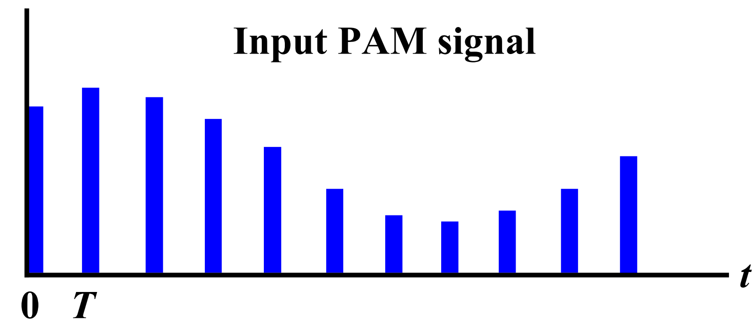


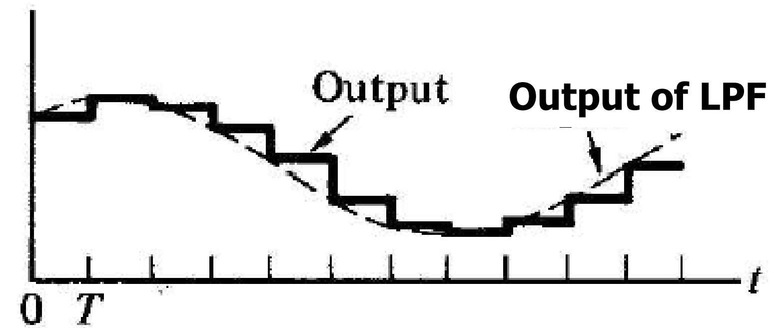
- Time-division multiplex (TDM) (continued)
  - Sample-and-hold circuit



$rC << \tau$: fast charge time

$RC >> T$: slow discharge time





- The switch closes when that particular channel is to be sampled.
- The capacitor changes abruptly to the input voltage within the time τ that the switch is closed.
- The capacitor retains the voltage level until next sample pulse.

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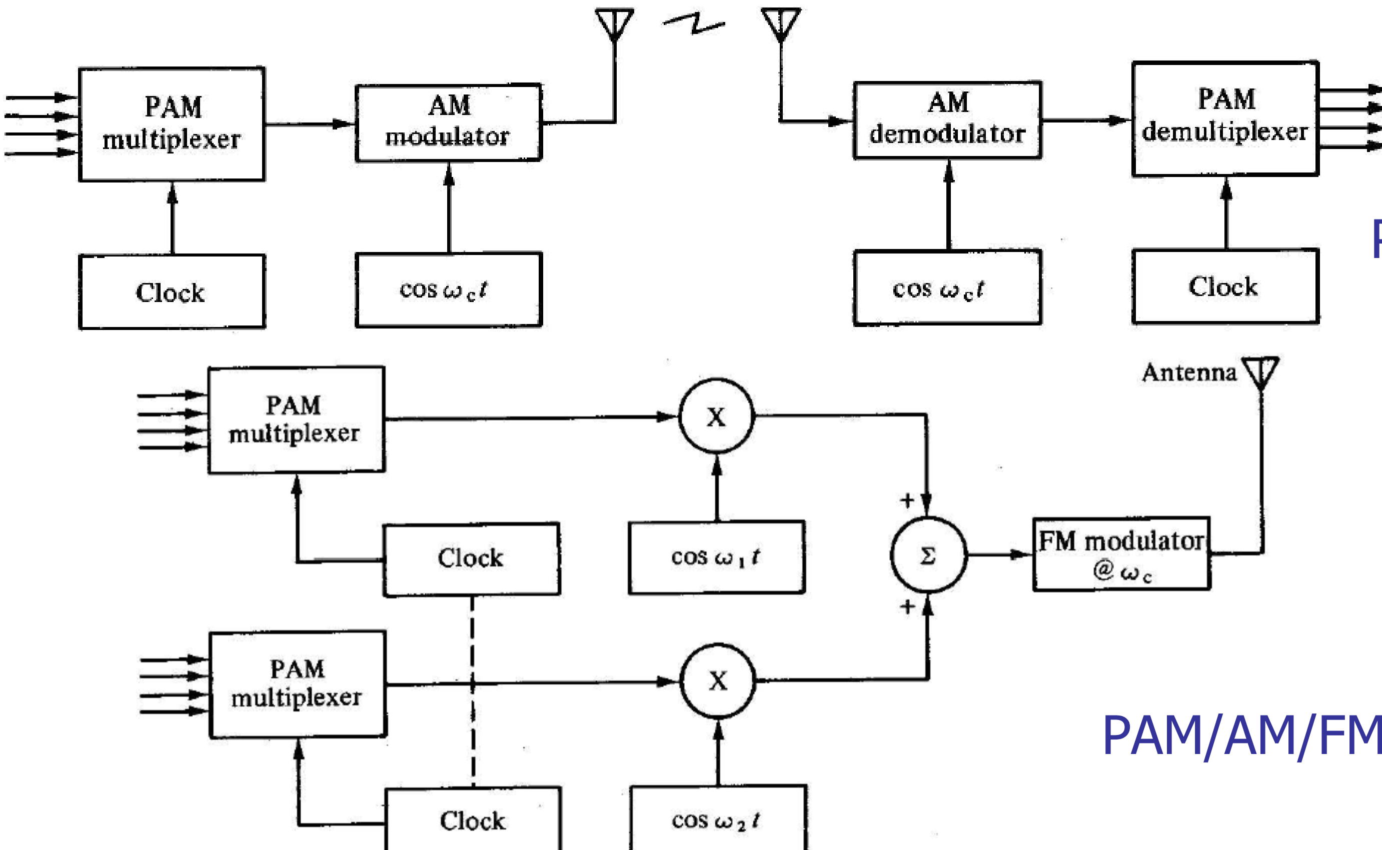
- Time-division multiplex (TDM) (continued)

  The principles of TDM apply to other types of pulse modulation.

  - Advantages of TDM
    - The circuitry required is digital, thus affording high reliability and efficient operation. Besides, it is simpler than the modulators and demodulators required in frequency-division multiplexing (FDM).
    - When the inputs are all of comparable bandwidths, TDM can multiplex many channels of low-frequency data very efficiently.
    - Comparing to FDM, the interchannel cross-talk arising from the nonlinearities of the amplifiers in TDM systems is relatively small.
  - Disadvantages of TDM
    - In TDM systems, pulse accuracy and timing jitter become a major problem at high frequencies so that TDM systems normally operate at clock frequencies below 100 MHz.
    - Accurate time synchronization is required between transmitter and receiver.

Chapter 7: Pulse Modulation

- Combine TDM and CW modulation
  - For long-distance transmission, continuous wave (CW) modulation may be used to translate the PAM spectrum to higher frequencies.



PAM/AM

PAM/AM/FM